Probabilistic models for image processing: applications in vascular surgery

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Thesis defense

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Outline

1. Introduction
2. Pairwise and Triplet Markov Models
3. More general probabilistic models
4. Applications to vascular surgery
5. Conclusion

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1. **Introduction**
   - Medical context
   - Probabilistic modeling
   - Hidden Markov Models
2. Pairwise and Triplet Markov Models
3. More general probabilistic models
4. Applications to vascular surgery
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Medical context

Cardiovascular diseases

- Human and monetary cost for society
  → ≈ 18 millions death worldwide in 2016 (31% of all the year deaths)[1]
  → Total cost estimated to 210 billions € in 2015 in Europe[2]

[1] https://www.who.int
[2] http://www.ehnheart.org/
[3] https://healthmetrics.heart.org/wp-content/uploads/2017/10/Cardiovascular-Disease-A-Costly-Burden.pdf

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- Increasing number of affected people: these are diseases linked to old age, sedentarity and bad life hygiene (nutrition, tobacco, ...)[3]

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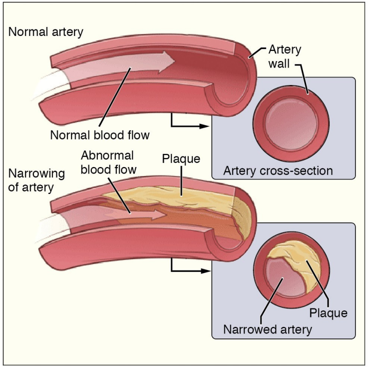
- Hypertension, strokes, myocardal infarcts, *etc.* → underlying pathology of the arteries: **the atherosclerosis**



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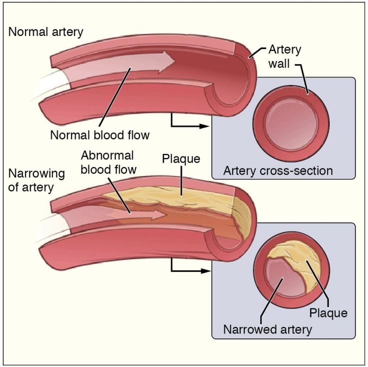

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- **Atheromateous plaques** are composed of calcium, lipids, macrophage cells, ...

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  → The patient does not need to be opened: **mini-invasive** and **image guided**
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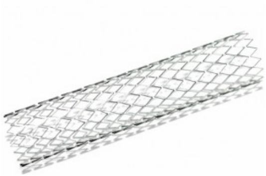
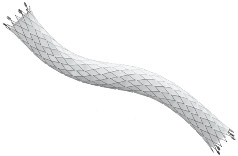


stent
www.cookmedical.com



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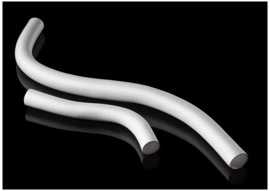

vascular prothesis
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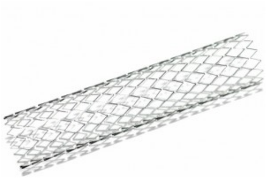

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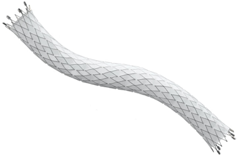

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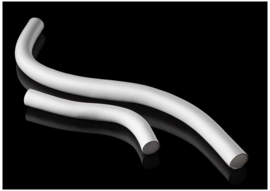

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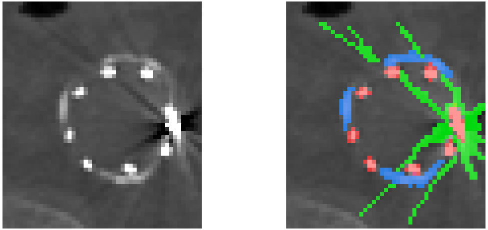
But biomaterials are recent and **not well understood**!

The data

CT: Computed Tomography / **microCT**

Example of input data (2D views)



Calcifications Stent Artifacts

Being able to segment such images could help develop the knowledge of biomaterials!

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→ **Missing tools for research on biomaterials**

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- **Sparse models** → fast and exact computations → hugeness of medical data
- **Dense models** → approximating methods → model very complex phenomena
- Combined with deep learning → many **top current results** in medical imaging

Probabilistic modeling

Probabilistic graphical models: the undirected case

- Let $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ be an **undirected** graph
  - $\mathcal{S}$ is the set of *sites*, *nodes* or *vertices*
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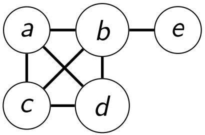
- $\mathcal{C}$ is the set of cliques of $\mathcal{S}$



An undirected graph

$\mathcal{S} = \{a, b, c, d, e\}$
$c = \{a, b, c, d\}$ is a clique
c is a fully connected subset
$\mathcal{N}_a = \{b, c, d\}$

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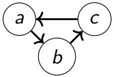
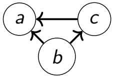


$\{a, b, c\}$ *is a cycle*



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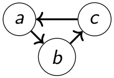

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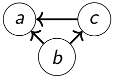

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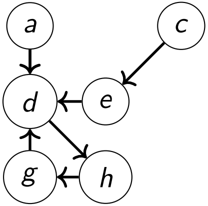

A directed graph

$\mathcal{S} = \{a, c, d, e, g, h\}$
$\bar{\mathcal{S}} = \{d, e, g, h\}$
a and c are root nodes
$\mathcal{P}(d) = \{a, e, g\}$

Probabilistic graphical models: random variables and graphs

- Associate a random variable X_s (or a random vector...) at every site s of $\mathcal{G}$
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- Segmentation criteria: $\rightarrow$ Maximum A Posteriori (MAP)
  $\rightarrow$ Maximum Posterior Mode (MPM)

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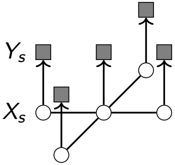
  - Generative models $\rightarrow p(\boldsymbol{x}, \boldsymbol{y})$ is modeled
  - $\boldsymbol{X}$ is a Markovian process and $p(\boldsymbol{y}|\boldsymbol{x}) = \prod_{s \in \mathcal{S}} p(y_s|x_s)$

Hidden Markov Models: classical models

Hidden Markov Field (HMF) (Geman et al. 1984)



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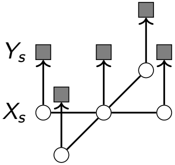

$p(\boldsymbol{x})$ is a Markov field

$$p(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{Z} \prod_{s \in \mathcal{S}} \tilde{p}(x_s | \boldsymbol{x}_{\mathcal{N}_s}) \tilde{p}(y_s | x_s)$$

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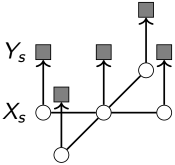

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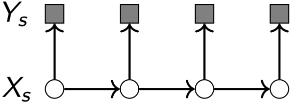

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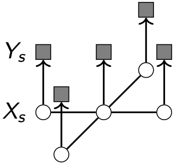

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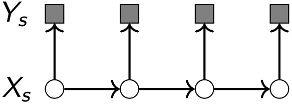

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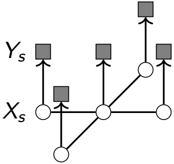

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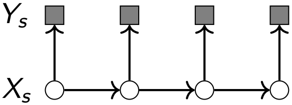

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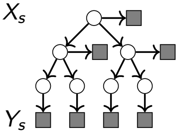

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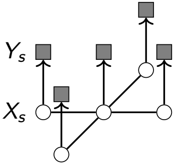

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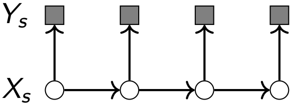

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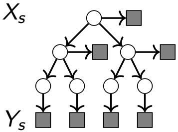

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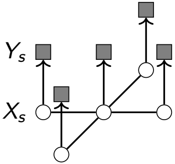

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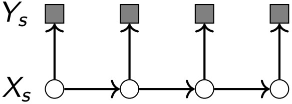

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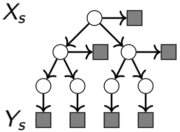

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Outline

1. Introduction
2. **Pairwise and Triplet Markov Models**
   - Extension of Hidden Markov Models
   - Gaussian Pairwise Markov Fields
   - Spatial Triplet Markov Trees
3. More general probabilistic models
4. Applications to vascular surgery
5. Conclusion

Extension of Hidden Markov Models

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Pairwise and triplet assumptions

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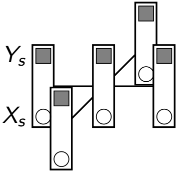

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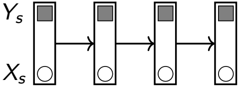

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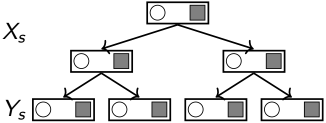

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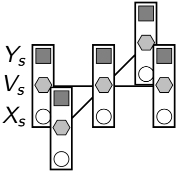

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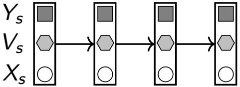
Triplet Markov Chain (Lanchantin et al. 2008)



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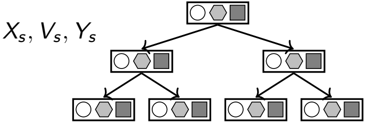
Triplet Markov Tree (TMT) (Courbot et al. 2018)



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    $\rightarrow$ Original hidden states:

$$p(\boldsymbol{x}|\boldsymbol{y}) = \sum_{\boldsymbol{v}} p(\boldsymbol{x},\boldsymbol{v}|\boldsymbol{y})$$

Gaussian Pairwise Markov Fields

Gaussian Pairwise Markov Fields: motivation

- **Gaussian Markov Random Fields** (GMRF) $\rightarrow$ a model for correlated noise

$$p(\boldsymbol{y}) = \frac{\exp\left(\frac{1}{2}\boldsymbol{y}^T Q \boldsymbol{y}\right)}{\sqrt{(2\pi)^N \det(Q^{-1})}},$$

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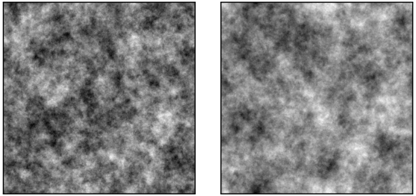

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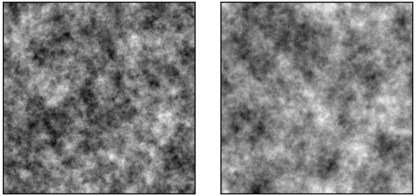

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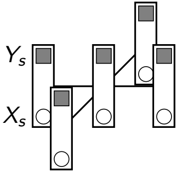
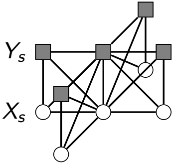
- PMF models can integrate GMRFs:



All possible direct dependencies
$\longrightarrow$



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- GPMFs are introduced in (Gangloff et al., submitted)

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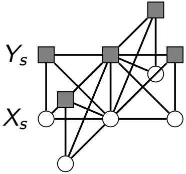

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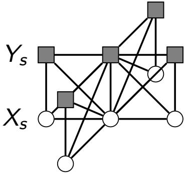

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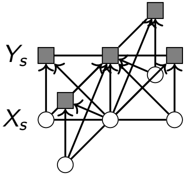
- Fewer direct dependencies in Potts-GMRFs:



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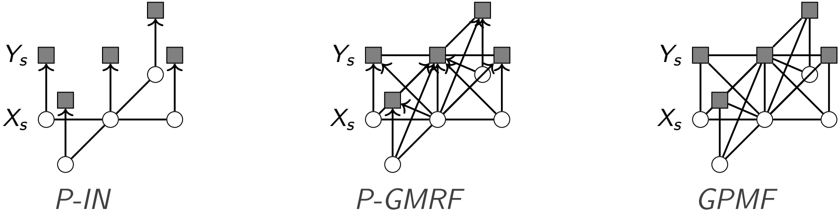
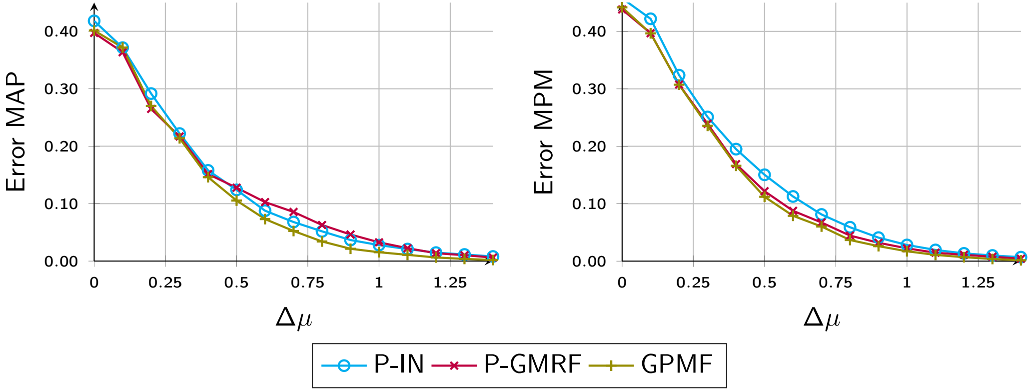


GPMF models: numerical applications



Error rate in segmentation for varying correlated noise levels

$\rightarrow$ The GPMF model always gives the best results

See synthetic images

Spatial Triplet Markov Trees

Spatial Triplet Markov Trees (STMTs)

Distribution of STMTs (Gangloff et al. 2020) (Gangloff et al., submitted):

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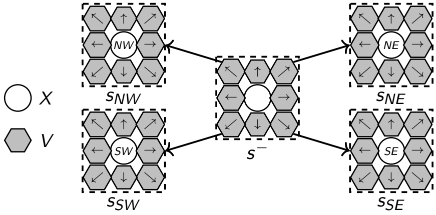

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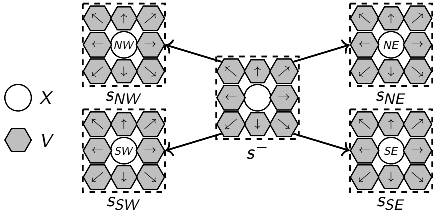
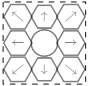


- We consider only observations Y_s at the finer resolution.

Designing the auxiliary process in STMTs



Propagation of spatial information: the same color indicates the same probability law

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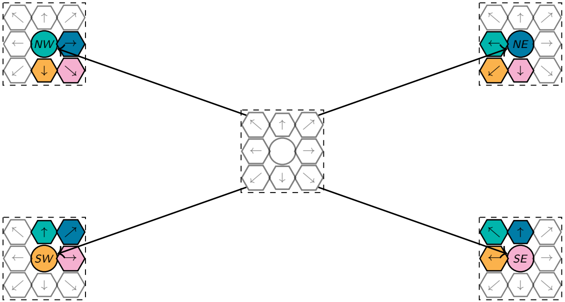

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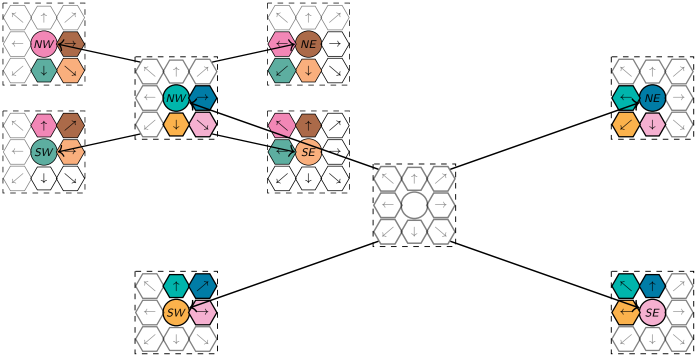

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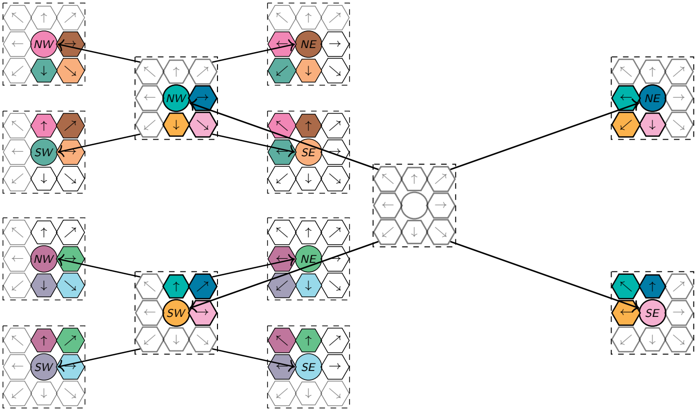

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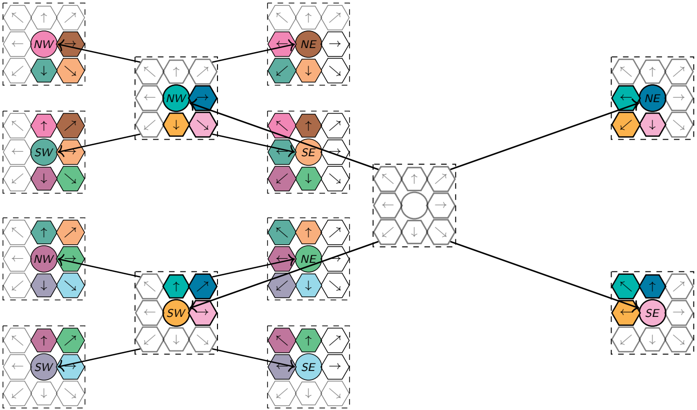

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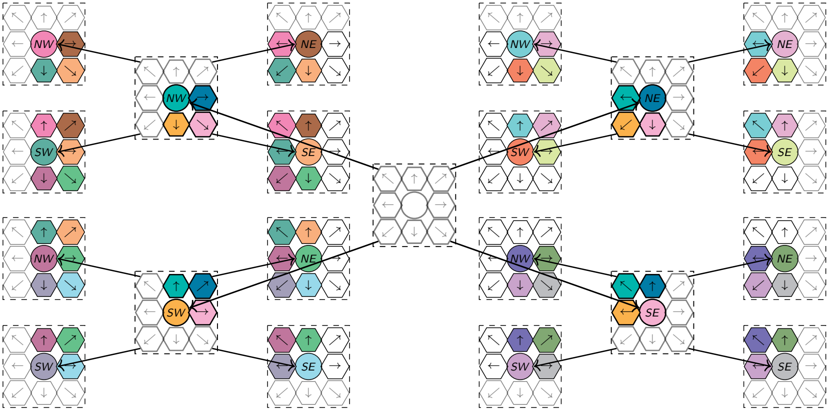

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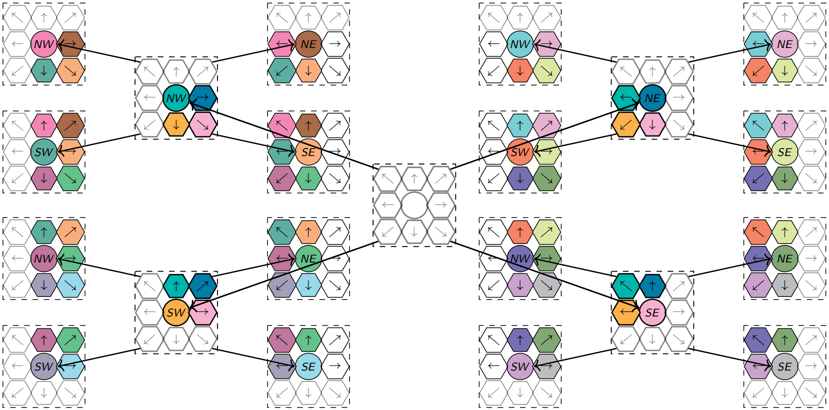

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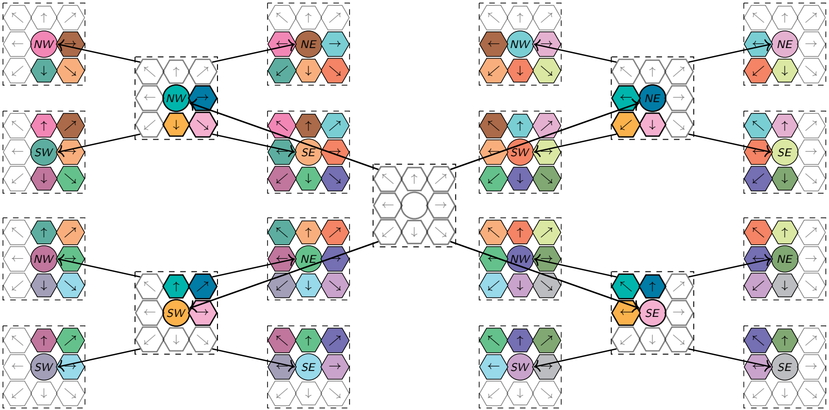

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STMTs: numerical applications

- Ising-like potentials to propagate spatial homogeneity similarly to Markov fields:

$$p(x_s|x_{s^-}, \boldsymbol{v}_{s^-}) = \frac{1}{Z} \exp\left(\alpha \delta_{x_s}^{x_{s^-}} + \sum_{v_{s^-} \in \boldsymbol{v}_{s^-}} \beta \delta_{x_s}^{v_{s^-}}\right), \text{ with } (\alpha, \beta) \in \mathbb{R}_+^2.$$

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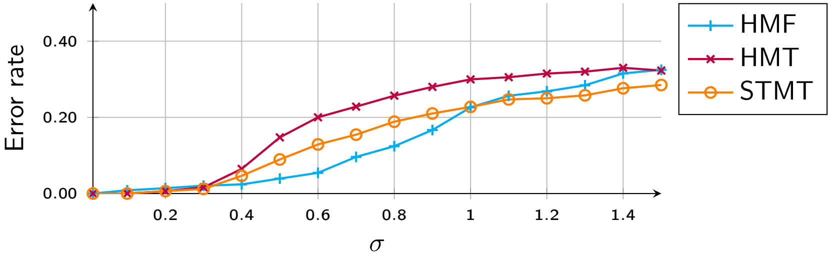
- Comparing HMFs, HMTs and STMTs in unsupervised segmentation:



Error rate in unsupervised segmentation function of the noise level

→ STMTs greatly improve HMT results

To conclude the section

Pairwise and Triplet Markov Models

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Outline

1. Introduction

2. Pairwise and Triplet Markov Models

3. **More general probabilistic models**
   - Going beyond Hidden Markov Models
   - Spatial Bayes Networks (SBNs)
   - Gaussian fully-connected Conditional Random Fields (fcCRFs)

4. Applications to vascular surgery

5. Conclusion

Going beyond Hidden Markov Models

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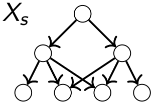

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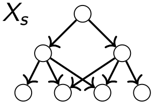

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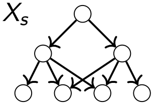

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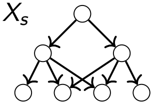

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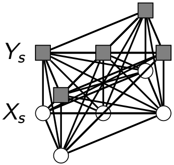

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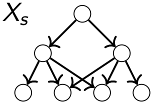

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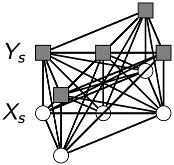

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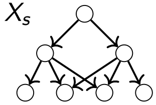

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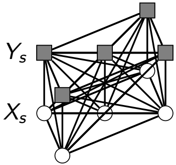

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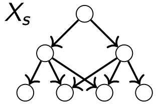

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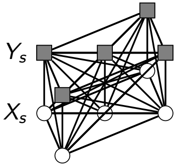

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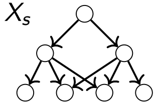

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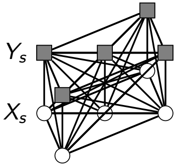

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- ... that (in general) approximate the results

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Variational Inference (VI) (Blei et al. 2017) $\rightarrow$ approximate inference when the true posterior $p(\boldsymbol{x}|\boldsymbol{y})$ is intractable

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$\rightarrow$ We now study the importance of choosing a rich family $\mathcal{Q}$

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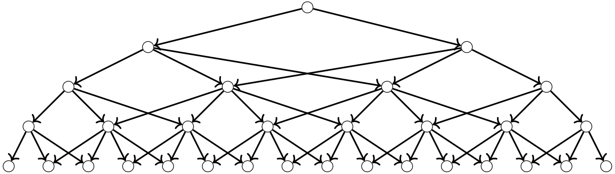

Graphical model for the SBN

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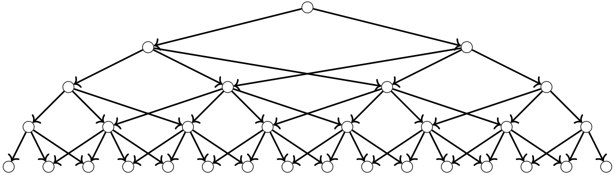
- The SBN model has for joint distribution:

$$p(\boldsymbol{x}) = p(x_r) \prod_{s \in \bar{\mathcal{S}}} p(x_s | x_{s^-}, x_{v(s)}), \text{ with } v: s \longmapsto \begin{cases} (s^-)^{\leftarrow} & \text{if } s \text{ is a left node,} \\ (s^-)^{\rightarrow} & \text{if } s \text{ is a right node.} \end{cases}$$

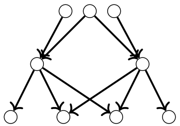


Graphical model for the SBN

- Inference will be carried with 3 different VIs (Gangloff et al. 2020)

Variational Inference in SBNs

Let us consider a small toy SBN
$\rightarrow p(\boldsymbol{x})$ is the target distribution



Target p
(indirect computations)

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- Approximation with Mean Field assumption:

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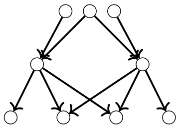

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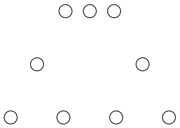

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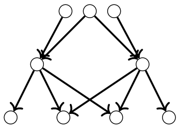

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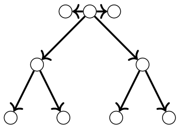

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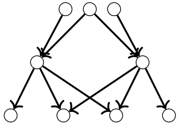
- Approximation with STMTs:

$$q^{STMT}(\boldsymbol{x}, \boldsymbol{v}) = q(x_r, v_r) \prod_{s \in \bar{\mathcal{S}}} q(x_s, v_s | x_{s^-}, v_{s^-})$$



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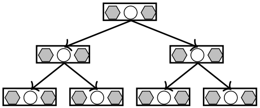

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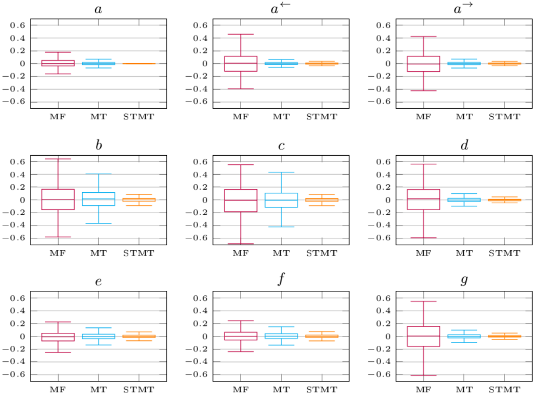
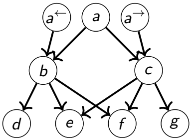
Variational Inference in SBNs





Target p

Dispersions of errors for true marginals estimation (1000 trials)

- Error dispersion: **MF VI** > **MT VI** > **STMT VI**

→ STMTs seem to **best capture the enhanced correlations** of SBNs

Gaussian fully-connected Conditional Random Fields (fcCRFs)

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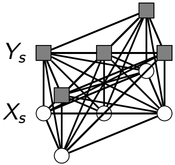
- **Discriminative model** $\rightarrow$ the posterior distribution is directly formulated:

$$p(\mathbf{x}|\mathbf{y}) = \frac{1}{Z} \exp\left(- \left(\sum_{s \in \mathcal{S}} \psi_u(x_s) + \sum_{(s,s') \in \mathcal{S}^2} (1 - \delta_{x_s}^{x_{s'}}) \sum_{r=1}^{2} w_r k_r(\mathbf{f}_s, \mathbf{f}_{s'}) \right) \right),$$

where | k_1 is a bilateral filtering kernel,
k_2 is a Gaussian kernel,
ψ_u are unary potentials.



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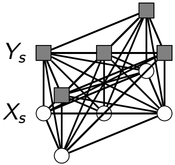

fcCRF

- This posterior is **intractable**: $\forall s \in \mathcal{S}$, $\mathcal{N}_s = \mathcal{S} \setminus \{s\}$

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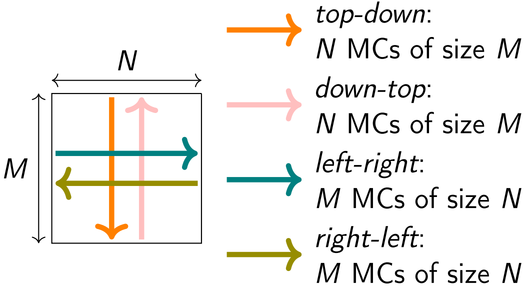

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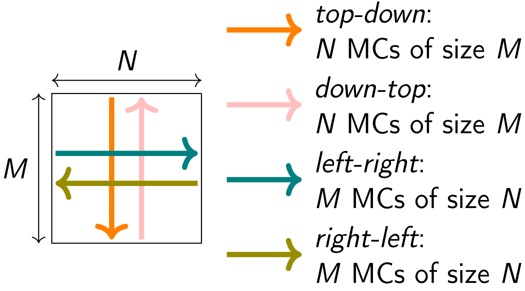
→ If N independent MCs of size M:

$$q(\boldsymbol{x}) = \prod_{n=1}^{N} q_1^n(x_1^n) \prod_{m=2}^{M} q_m^n(x_m^n | x_{m-1}^n)$$



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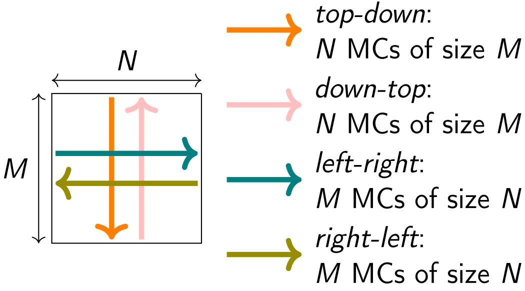
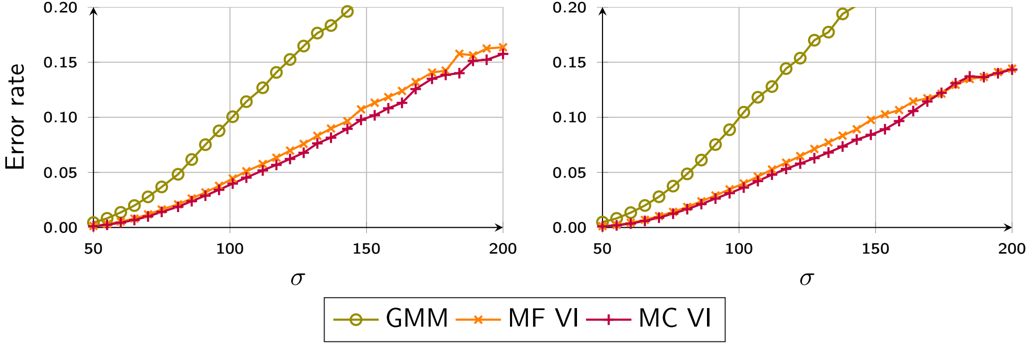


Remark: The idea is similar to that of Factorial HMMs (Ghahramani et al. 1997)

Variational Inference in fcCRFs: numerical applications

Remark: A Gaussian Mixture Model (GMM) is used to initialize the fcCRF model



Error rate as a function of σ (two different ranges)

$\rightarrow$ MC VI gives a few point improvement for a small additional computational cost

▸ See synthetic images

To conclude the section

More general probabilistic models

- Inference has become much more complex

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More general probabilistic models

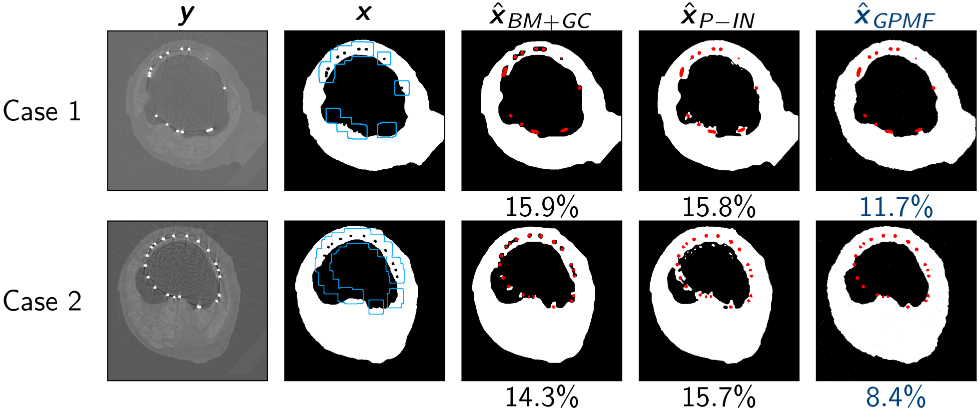
- Inference has become much more complex
- VI as a way to approximate the intractable posterior
- Importance of the choice of the variational distribution

Outline

1. Introduction
2. Pairwise and Triplet Markov Models
3. More general probabilistic models
4. **Applications to vascular surgery**
   - Segmentations of degraded images
   - Histological segmentations
5. Conclusion

Segmentations of degraded images

Segmentation of organic biomaterial with artifacts



Unsupervised segmentations of organic material in corrupted X-rays images

→ Best overall segmentation score for GPMF classifications

Segmentation of organic biomaterial with artifacts

	FN	FP
BM3D+GC	0.14	**0.01**
P-IN	**0.08**	0.08
GPMF	**0.08**	0.04

(a) Case 1

	FN	FP
BM3D+GC	0.05	**0.07**
P-IN	**0.02**	0.14
GPMF	**0.02**	**0.07**

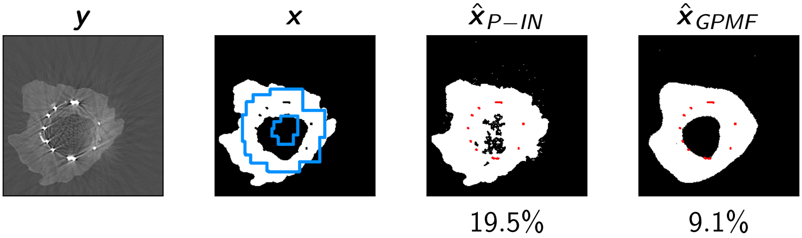
(b) Case 2

Table: FN and FP rates in corrupted areas

→ Best False Positive / False Negative compromise for GPMF

Segmentation of organic biomaterial with artifacts

⚠ Smoothing effect can lead to spurious classifications



GPMF segmentations (limiting cases)

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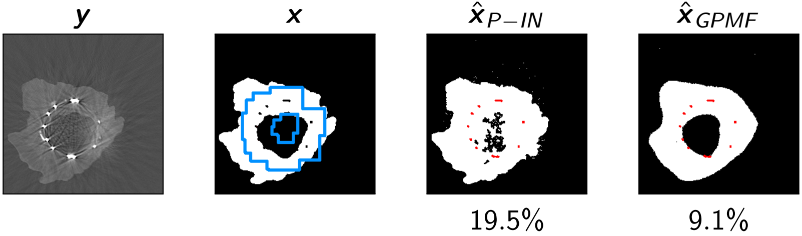

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- Probably due to the assumed stationarity of the noise range and strength

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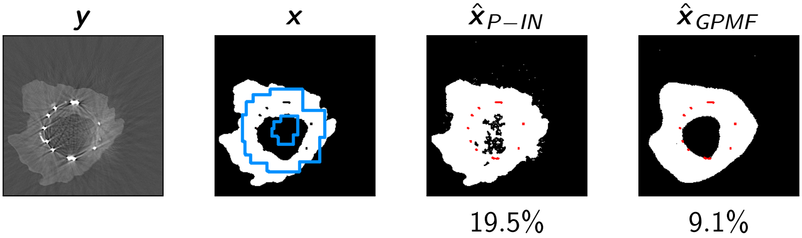

GPMF segmentations (limiting cases)

- Probably due to the assumed stationarity of the noise range and strength
- Introduce non-stationarity (*e.g.* with triplet Markov model (Lanchantin et al. 2008))

Histological segmentations

3D histological segmentations

The goal is to perform 3D segmentations with histological classes of microCT

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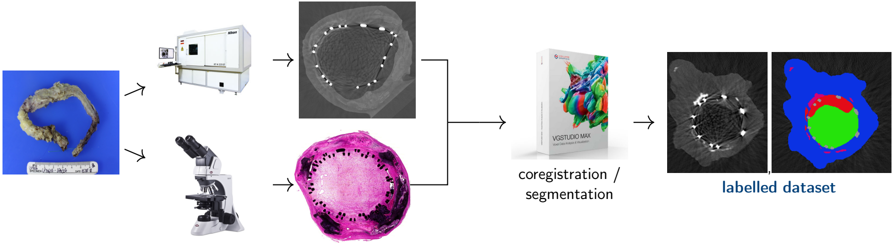
- Construction of the protocol and of the dataset



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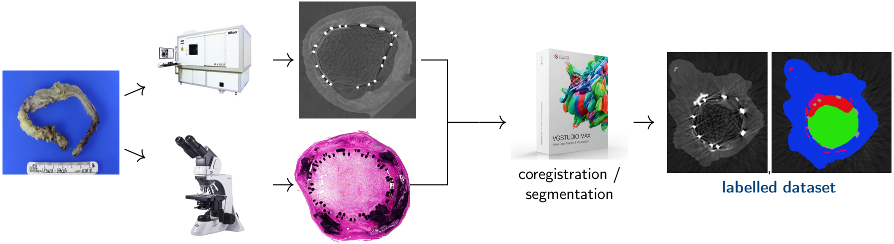

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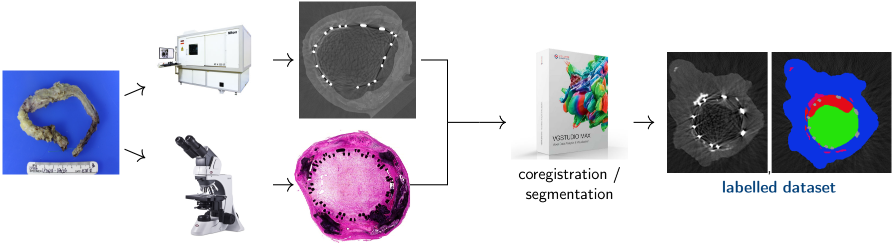
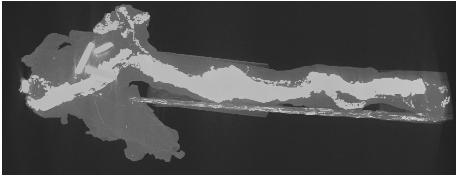
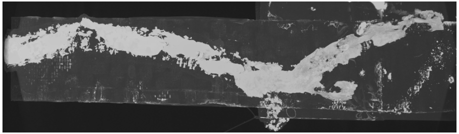


- 6 histological classes of interest: *background, sheet and nodular calcifications, soft tissues, fatty tissues, specimen holder*
- Convolutional Neural Network + fcCRF for **3D segmentations**

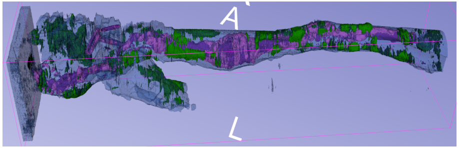
3D histological segmentations

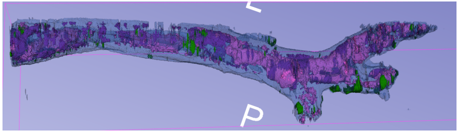
Original images





Segmentations





Data cubes of grayscale mCTs and histological segmentations of explanted arteries

(Gangloff et al., submitted)

3D histological segmentations

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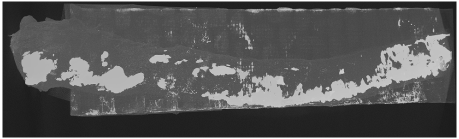

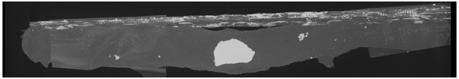

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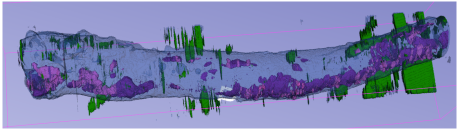

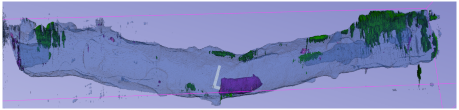

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To conclude the section

Applications to vascular surgery

- Fine segmentation of the stent and its environment

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Applications to vascular surgery

- Fine segmentation of the stent and its environment
- First histologic segmentation of explants combining deep learning and probabilistic graphical models

Outline

1. Introduction
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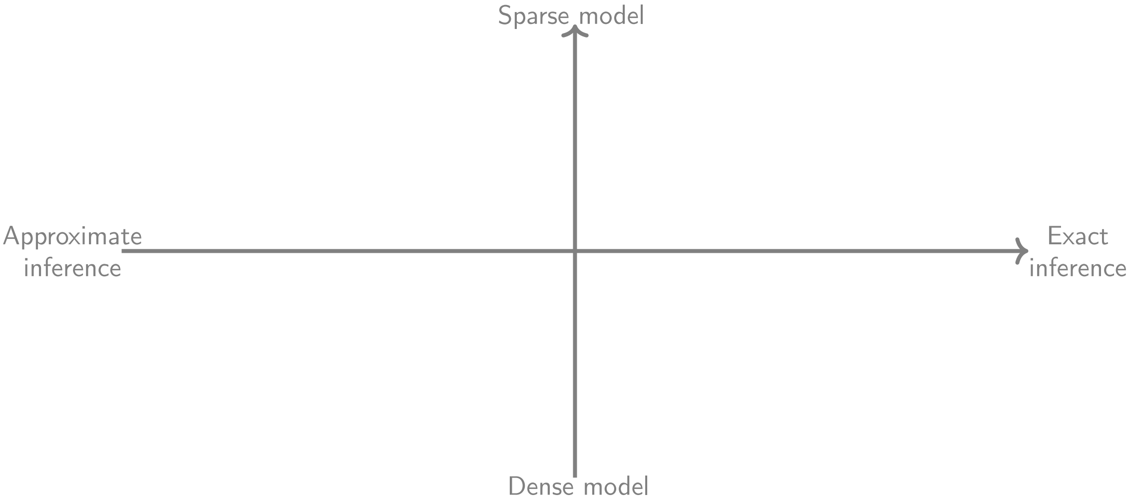
4. Applications to vascular surgery
5. **Conclusion**
   - Conclusions & Perspectives
   - Publications

Conclusions & Perspectives

Summary of the models



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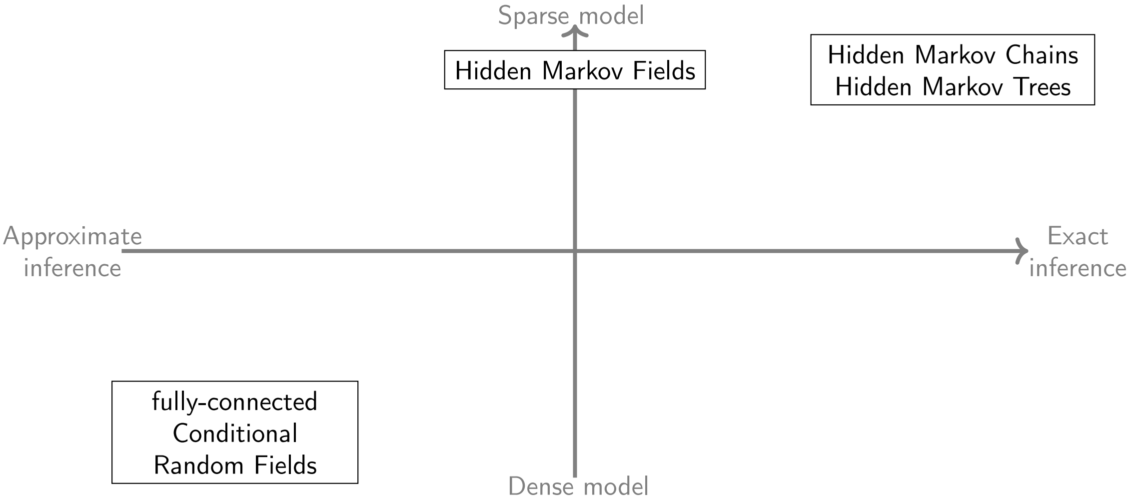

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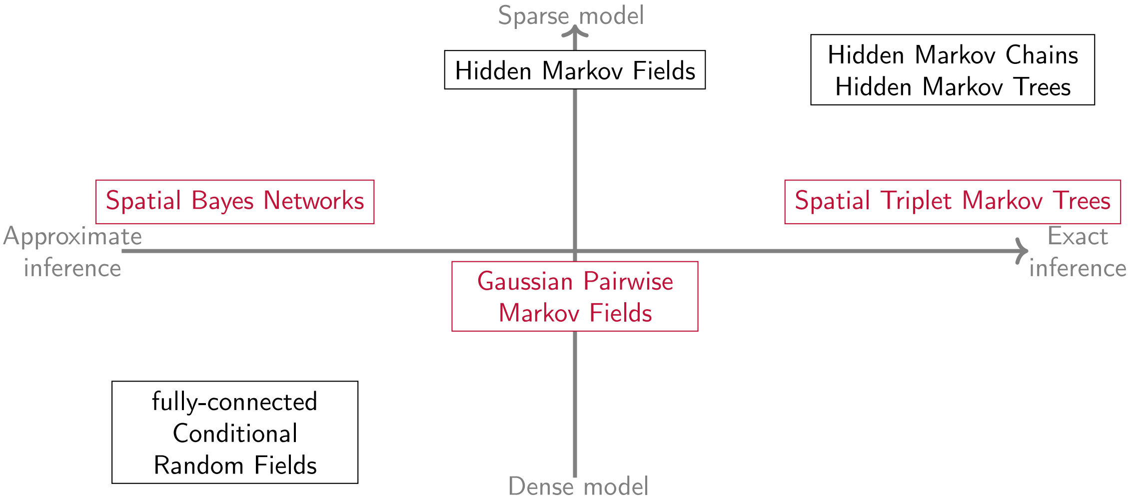

Conclusions & Perspectives

Probabilistic graphical models:

- A field where theory and applications well complement each other

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- Much is yet to discover on biomaterials and the diseases
- Improve the treatments and the prevention

Publications

Publications

Journal

- Unsupervised Segmentation with Gaussian Pairwise Markov Fields, H. Gangloff, J.-B. Courbot, E. Monfrini, C. Collet, submitted to *CSDA*
- Automated histological segmentation on micro-computed tomography images of atherosclerotic arteries, S. Kuntz, H. Gangloff, H. Naamoune, E. Monfrini, C. Collet, A. Lejay, M. Kutyna, R. Virmani, N. Chakfé, submitted to *EJVES*
- Co-registration of peripheral atherosclerotic plaques assessed by conventional CT-angiography, micro-CT and histology in CLTI patients, S. Kuntz, H. Jinnouchi, M. Kutyna, S. Torii, A. Cornelissen, Y. Sato, M. E. Romero, F. Kolodgie, A. V. Finn, A. Schwein, M. Ohana, H. Gangloff, A. Lejay, N. Chakfé, R. Virmani, *EJVES*, 2020.
- Assessing the segmentation performance of pairwise and triplet Markov models, I. Gorynin, H. Gangloff, E. Monfrini, W. Pieczynski, *SP*, 2018.

Conference

- Unsupervised Image Segmentation with Spatial Triplet Markov Trees, H. Gangloff, J.-B. Courbot, E. Monfrini, C. Collet, submitted to *ICASSP*, 2021.
- Markov Chain Variational Inference in Fully-Connected Conditional Random Fields, H. Gangloff, E. Monfrini, C. Collet, submitted to *ICASSP*, 2021.
- Improved Centerline Tracking for new descriptors of atherosclerotic aortas, H. Gangloff, E. Monfrini, M.Z. Ghariani, M. Ohana, C. Collet, N. Chakfé, *IPTA*, 2020.
- Unsupervised segmentation of stents corrupted by artifacts in medical X-ray images, H. Gangloff, E. Monfrini, C. Collet, N. Chakfé, *IPTA*, 2020.
- Spatial Triplet Markov Trees for auxiliary variational inference in Spatial Bayes Networks, H. Gangloff, J.-B. Courbot, E. Monfrini, C. Collet, *SMTDA*, 2020.
- Segmentation de stents dans des données médicales à rayons-X corrompues par les artéfacts, H. Gangloff, E. Monfrini, C. Collet, N. Chakfé, *GRETSI*, 2019.
- Segmentation non-supervisée dans les champs de Markov couples gaussiens, H. Gangloff, J.-B. Courbot, E. Monfrini, C. Collet, *GRETSI*, 2019.
- Performance comparison across hidden, pairwise and triplet Markov models' estimators, I. Gorynin, L. Crelier, H. Gangloff, E. Monfrini, W. Pieczynski, *ICACM*, 2016.

GPMF: Proof of the distribution

- **Necessity:** Using $p(\boldsymbol{y}|\boldsymbol{x}) = \frac{p(\boldsymbol{x},\boldsymbol{y})}{\int_{\mathbb{R}^N} \mathrm{d}\boldsymbol{y}\, p(\boldsymbol{x},\boldsymbol{y})}$ where:

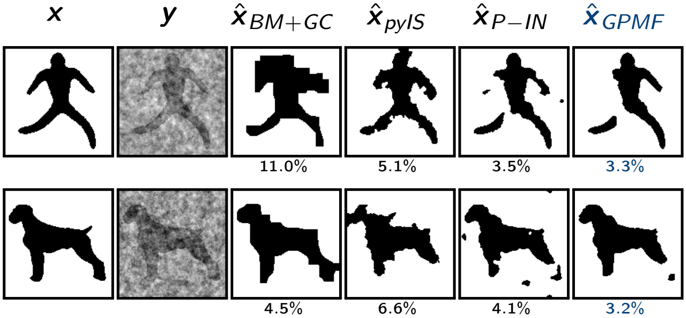
$$p(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{Z}\exp\left(-\sum_{n=1}^{|\mathcal{N}|}\left(\sum_{\boldsymbol{c}\in\mathcal{C}_n} V_n(\boldsymbol{x_c},\boldsymbol{y_c})\right)\right) \text{ and } p(\boldsymbol{y}|\boldsymbol{x}) = \frac{\exp\left(-\sum_{s,s'\in\mathcal{S}^2} y_s C_{s,s'} y_{s'}\right)}{\sqrt{2\pi \det(C^{-1})}},$$

  where C is a SPD matrix. By equivalences, we get the constraints on V_n.

- **Sufficiency:**
  - **PMF** w.r.t. $\mathcal{N}$: $p(\boldsymbol{x},\boldsymbol{y}) > 0, \forall \boldsymbol{x} \in \Omega^N, \forall \boldsymbol{y} \in \mathbb{R}^N$ and $\forall s \in \mathcal{S}$, $p(x_s, y_s|\boldsymbol{x}_{\mathcal{S}\setminus s}, \boldsymbol{y}_{\mathcal{S}\setminus s}) = p(x_s, y_s|\boldsymbol{x}_{\mathcal{N}_s}, \boldsymbol{y}_{\mathcal{N}_s})$.
  - $p(\boldsymbol{y}|\boldsymbol{x})$ **is a GMRF:** We develop $p(\boldsymbol{y}|\boldsymbol{x}) = \frac{\exp(-E(\boldsymbol{x},\boldsymbol{y}))}{\int_{\mathbb{R}^N} \mathrm{d}\boldsymbol{y}\, \exp(-E(\boldsymbol{x},\boldsymbol{y}))}$ to get the result by using $E(\boldsymbol{x},\boldsymbol{y}) = \sum_{n=1}^{2}\sum_{\boldsymbol{c}\in\mathcal{C}_n} \bar{V}_n(\boldsymbol{y_c},\boldsymbol{x_c}) + \sum_{n=1}^{|\mathcal{N}|}\sum_{\boldsymbol{c}\in\mathcal{C}_n} \tilde{V}_n(\boldsymbol{x_c})$,

▸ Back to GPMF definition

GPMF models: numerical applications



Unsupervised segmentation of images from the dataset

Remark: $\hat{x}_{pyIS}$ from (Borovec et al. 2017) and $\hat{x}_{BM+GC}$ from (Dabov et al. 2009)

▸ Back to GPMF numerical applications

GPMF time complexity

- Stochastic Parameter Estimation algorithm

For T SPE iterations $\rightarrow T(T+1)/2$ total Gibbs sampler runs

For $T = 30 \rightarrow 465$ Gibbs sampler runs ~ 120 seconds (with $r = 6$)

- Image segmentation

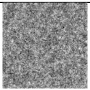
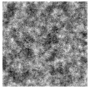
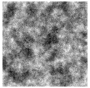
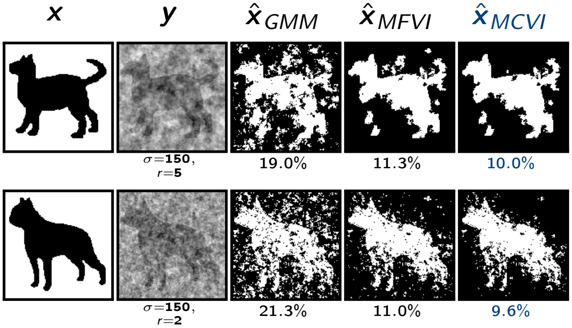
Range of correlations		P-IN	GPMF
	$\rightarrow r = 1$	15s	2min10s
	$\rightarrow r = 3$	15s	3min20s
	$\rightarrow r = 6$	15s	8min

Table: Time in MPM segmentation of a 130×130 image

Variational Inference in fcCRFs: numerical applications



Supervised segmentations of images corrupted

→ Using MC VI always leads to an improvement of a few points in the segmentation results of MF VI for very small additional computational cost

▸ Back to MC VI numerical applications

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