#3629

Unsupervised Image Segmentation with Spatial Triplet Markov Trees

Hugo Gangloff[1,2,3], Jean-Baptiste Courbot[4], Emmanuel Monfrini[1], Christophe Collet[2]

[1]Samovar, Télécom SudParis, Institut Polytechnique de Paris, Palaiseau, France
[2]ICube, Université de Strasbourg, CNRS UMR 7357, Strasbourg, France
[3]GEPROVAS, Strasbourg, France
[4]IRIMAS UR 7499, Université de Haute-Alsace, Mulhouse, France

46th ICASSP | June 06-11, 2021 | Toronto, Canada (virtual conference)

MOTIVATIONS

Probabilistic graphical models are widely used in unsupervised image segmentation. Direct dependencies between random variables are introduced to model potentially complex phenomena but many constraints exist to maintain the model tractability: such a compromise is successfully made in Hidden Markov Models. They are then a famous family of probabilistic graphical models [1].

OBJECTIVES

- Generalization of Markov Trees to enhance the spatial interactions for image segmentation: Spatial Triplet Markov Trees (STMTs).
- Exact and deterministic inference in the new model as in classical Hidden Markov Trees (HMTs) [2].
- Design a parameter estimation procedure for unsupervised segmentation.

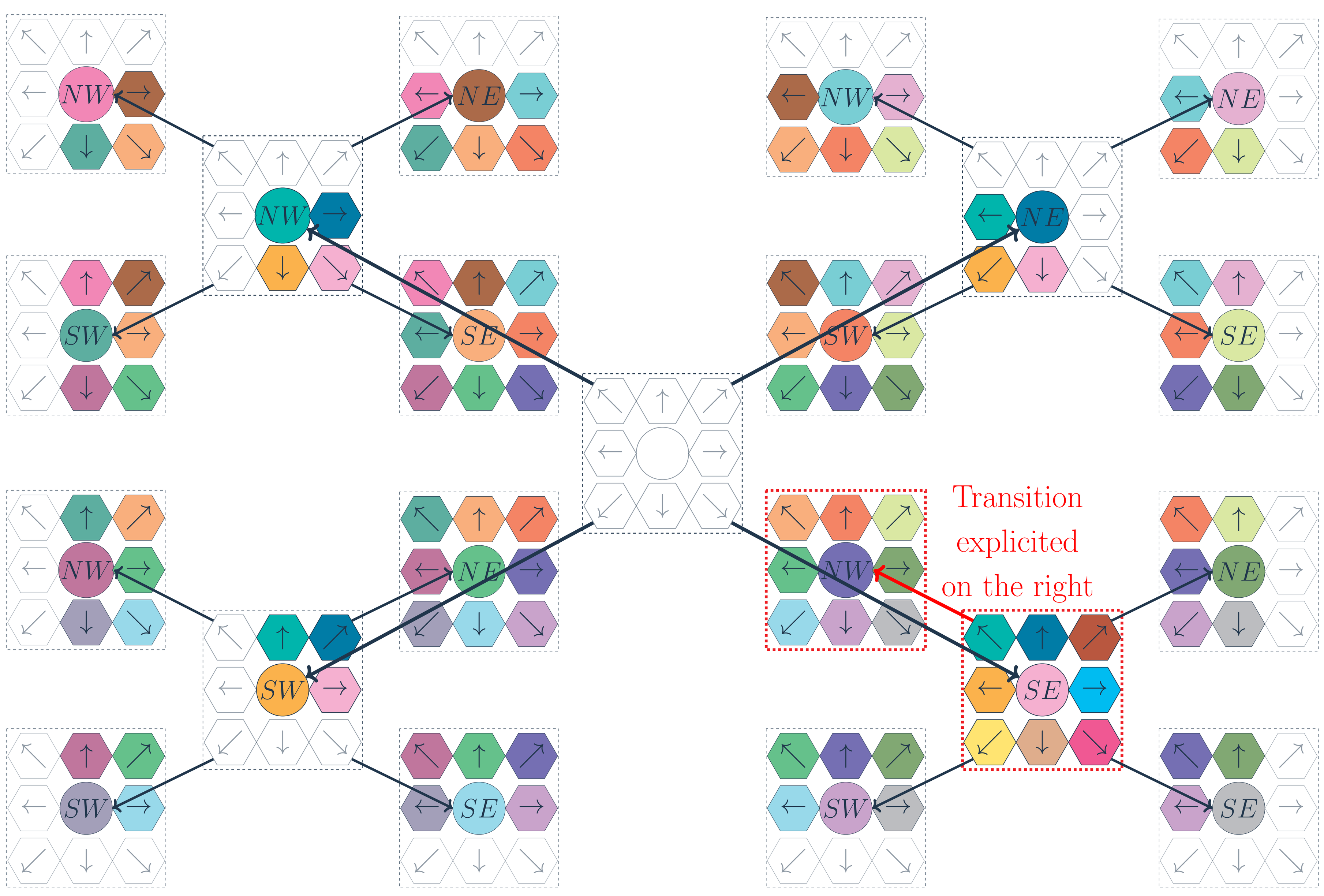
SPATIAL TRIPLET MARKOV TREES

$\boldsymbol{X}$ is a discrete hidden process, $\boldsymbol{V}$ is a discrete auxiliary process and $\boldsymbol{Y}$ is a continuous observed process. In the STMT model, at each site $s \in \mathcal{S}$, $\boldsymbol{V}_s$ is an octuplet, *i.e.*, $\boldsymbol{V}_s = (V^{\leftarrow}, V^{\nwarrow}, V^{\uparrow}, V^{\nearrow}, V^{\rightarrow}, V^{\searrow}, V^{\downarrow}, V^{\swarrow})$. The triplet $(\boldsymbol{X}, \boldsymbol{V}, \boldsymbol{Y})$ is a Markov tree with L resolutions:

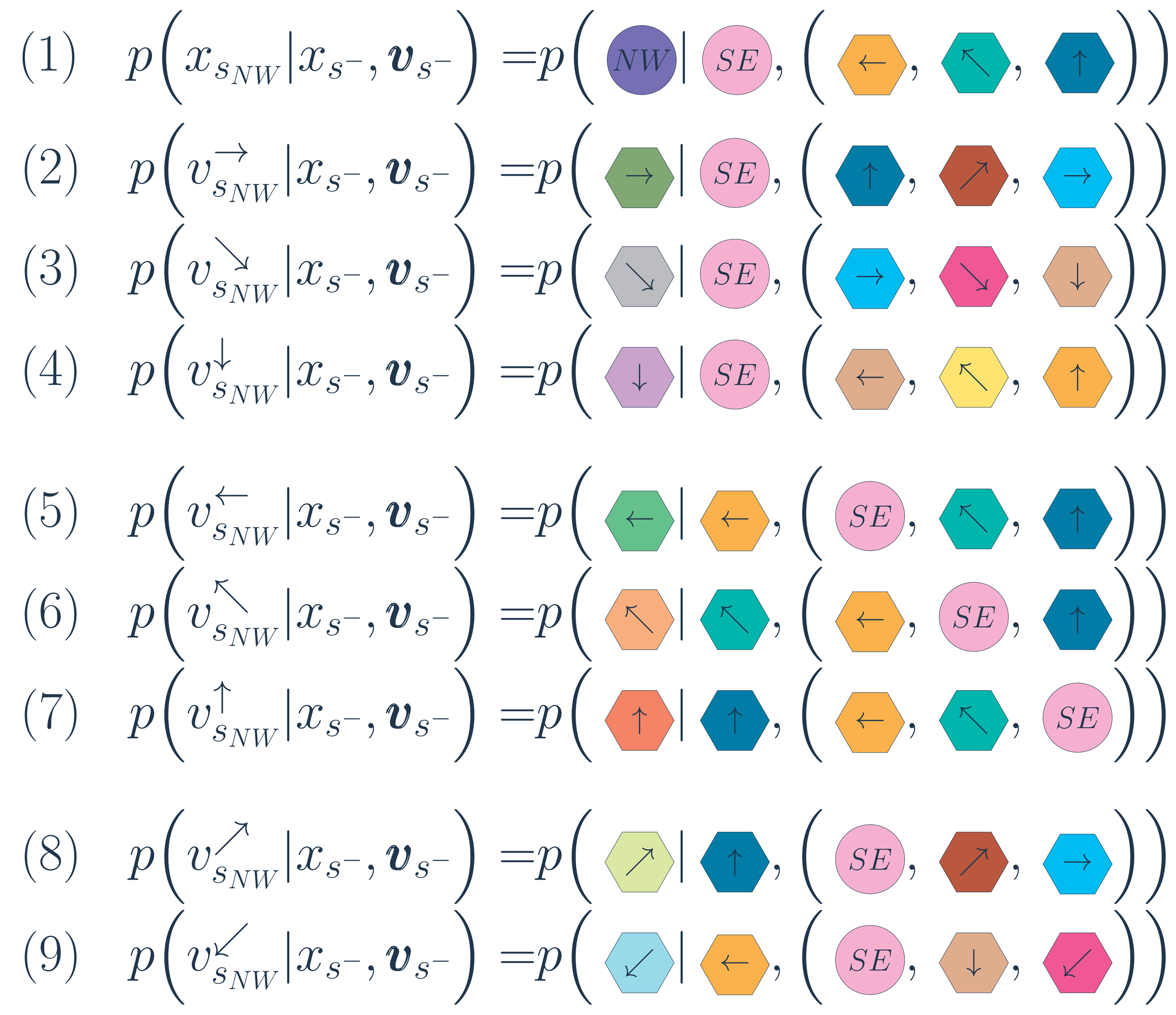
$$p(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{y}) = p(x_r, \boldsymbol{v}_r, y_r) \prod_{s \in \mathcal{S}\setminus\mathcal{S}^L} p(x_s, \boldsymbol{v}_s, y_s | x_{s^-}, \boldsymbol{v}_{s^-}, y_{s^-}), \text{ with, } \forall s \notin \mathcal{S}^L,$$

$$p(x_s, \boldsymbol{v}_s, y_s | x_{s^-}, \boldsymbol{v}_{s^-}, y_{s^-}) = \underset{(1)}{p(x_s|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(2)}{p(v_s^{\rightarrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(3)}{p(v_s^{\searrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(4)}{p(v_s^{\downarrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(5)}{p(v_s^{\leftarrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(6)}{p(v_s^{\nwarrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(7)}{p(v_s^{\uparrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(8)}{p(v_s^{\nearrow}|x_{s^-}, \boldsymbol{v}_{s^-})} \underset{(9)}{p(v_s^{\swarrow}|x_{s^-}, \boldsymbol{v}_{s^-})}$$

Note that $\forall s \notin \mathcal{S}^L$, y_s is not modeled and $\forall s \in \mathcal{S}^L$, $p(y_s|x_{s^-}, v_{s^-}, y_{s^-}, x_s, v_s) = p(y_s|x_s)$ (independent Gaussian noise).



Equations for the transition in red:



We introduce Ising-like transitions, $\forall s \in \mathcal{S}$:

$$p(x_s|x_{s^-}, \boldsymbol{v}_{s^-}) = \frac{1}{Z}\exp\left(\alpha\delta_{x_s}^{x_{s^-}} + \sum_{v_{s^-} \in \boldsymbol{v}_{s^-}} \beta\delta_{x_s}^{v_{s^-}}\right), (\alpha, \beta) \in \mathbb{R}_+^2.$$

Figure 1: The first three resolutions of a STMT: details on the modeling of the transitions. The sites where the random variables have the same law are represented with the same color. This then highlights the spatial homogeneity that is transmitted layer after layer: in the new model, a process X_s is sampled with information from the father's neighbors. The variables that remain uncolored do not serve our illustration.

PARAMETER ESTIMATION & INFERENCE

Parameter estimation: Stochastic Expectation-Maximization as in [3]:
→ Linear Least-Square estimator for α and β.
→ Maximum Likelihood estimator for the noise parameter.
Inference: Generalized Upward-Downward algorithm for Triplet Trees [2]:

$$\forall s \in \mathcal{S}, p(x_s|y_s) = \sum_{\boldsymbol{v}_s} p(x_s, \boldsymbol{v}_s|y_s) = \sum_{\boldsymbol{v}_s} \alpha_s(x_s, \boldsymbol{v}_s)\beta_s(x_s, \boldsymbol{v}_s),$$

where α_s and β_s are the upward and downward probabilities.

RESULTS

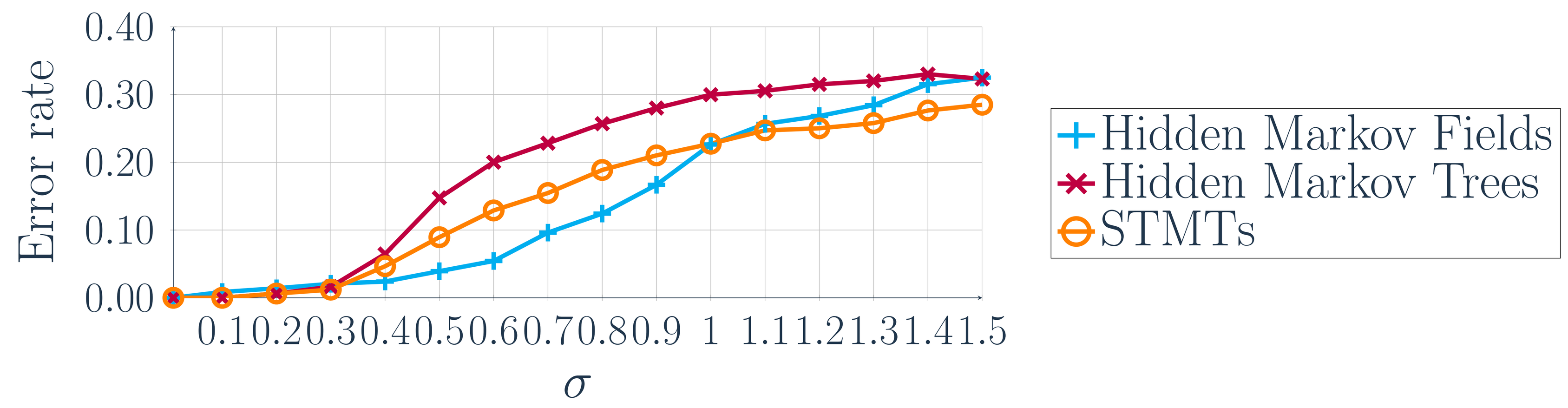


Figure 2: Error rate in unsupervised segmentation for varying noise level.
→ STMTs greatly improve HMT results and are closer to HMF results.

CONCLUSION

- STMTs outperform classical HMTs in unsupervised image segmentation: stronger spatial correlations with still an exact inference procedure.
- Study of the theoretical links between Markov fields and STMTs.
- Towards a deterministic counterpart of Markov fields ?

References

[1] I. Gorynin, H. Gangloff, E. Monfrini, W. Pieczynski, *Assessing the segmentation performance of Pairwise and Triplet Markov models*, Signal Processing, 2018.
[2] J.-M. Laferté, P. Pérez, F. Heitz, *Discrete Markov image modeling and inference on the quadtree*, IEEE Transactions on Image Processing, 2000.
[3] J.-B. Courbot, E. Monfrini, V. Mazet, C. Collet, *Triplet Markov Trees for image segmentation*, IEEE Workshop on Statistical Signal Processing. 2018.

s@movar ICUBE GEPROVAS hugo.gangloff@telecom-sudparis.eu